Year 7 Knowledge Organisers

White Rose Maths

Bishop Walsh Catholic School

Autumn Term 2
Algebraic Thinking

Unit 4: Algebraic Notation
Unit 5: Equality and Equivalence
Unit 6: Sequences

ALGEBRAIC THINKING...
Unit 4: Algebraic Notation
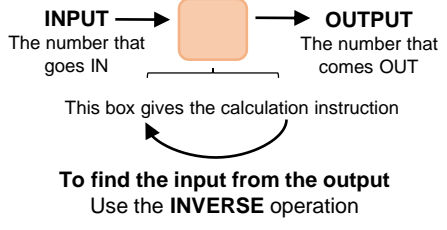
What do I need to be able to do?

By the end of this unit you should be able to:
- Be able to use letters to represent numbers
- Be able to substitute into single and two step function machines.
- Be able to use inverse operations
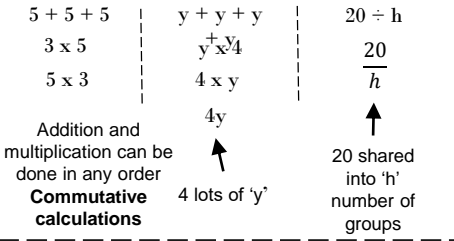- Find functions from expressions.
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Keywords

Function: a relationship that instructs how to get from an input to an output.
Input: the number/ symbol put into a function.
Output: the number/ expression that comes out of a function.
Operation: a mathematical process (add, subtract, times, divide etc)
Inverse: the operation that undoes what was done by the previous operation.
Commutative: the order of the operations do not matter.
Substitute: replace one variable with a number or new variable.
Expression: a maths sentence with numbers or letters but no equals sign
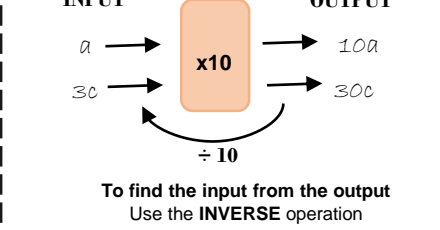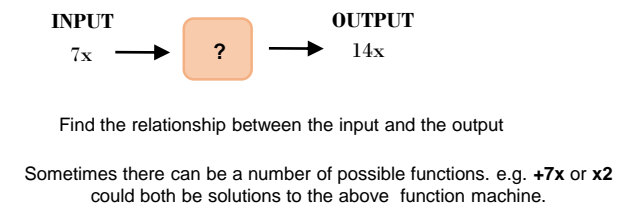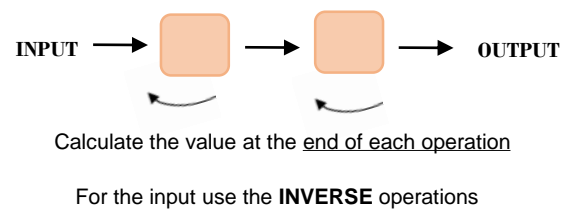**Evaluate:** work out

Single function machines

INPUT → [ ] → OUTPUT
The number that goes IN   The number that comes OUT

This box gives the calculation instruction

To find the input from the output
Use the **INVERSE** operation

Using letters to represent numbers

$5 + 5 + 5$	$y + y + y$	$20 \div h$
3×5	$y \times y \times 4$	$\dfrac{20}{h}$
5×3	$4 \times y$	
	$4y$	

Addition and multiplication can be done in any order
Commutative calculations

4 lots of 'y'

20 shared into 'h' number of groups

Single function machines (algebra)

INPUT → [x10] → OUTPUT
a → $10a$
$3c$ → $30c$
$\div 10$

To find the input from the output
Use the **INVERSE** operation

Find functions from expressions

INPUT → [?] → OUTPUT
$7x$ → $14x$

Find the relationship between the input and the output

Sometimes there can be a number of possible functions. e.g. **+7x** or **x2** could both be solutions to the above function machine.

Two step function machines

INPUT → [ ] → [ ] → OUTPUT

Calculate the value at the <u>end of each operation</u>

For the input use the **INVERSE** operations

Two step function machines (algebra)

b → [x 5] → $5b$ → [+ 4] → $5b + 4$

$\div 5$ $- 4$

c → [+ 4] → $c+4$ → [x 5] → $5(c + 4) = 5c + 20$

$- 4$ $\div 5$

IMPORTANT Calculate the value at the <u>end of each operation</u>

NOTE: The whole first output is multiplied by 5

Find functions from expressions

NOTE: the difference in the two expressions

$\dfrac{f + 5}{3}$

f add 5 then divide by 3

f → [x 5] → [+ 3]

$\dfrac{f + 5}{3}$

f divided by then add 5

f → [÷ 3] → [+ 5]

Sometimes it helps to try to explain the expression in words

Substitute into simple expressions

If $y = 7$

| $4y$ | INPUT → [X 4] → OUTPUT | $\begin{array}{c}4 \times 7 \\ \text{OR} \\ 7 \times 4\end{array}$ | = 28 |

| $y - 2$ | INPUT → [ ] → OUTPUT | 7 - 2 | = 5 |
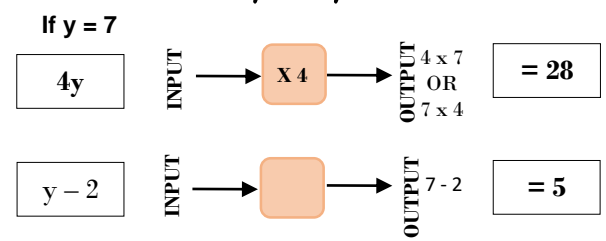
Substitute into more complex expressions

Put the expression into a function machine. X is the input.

If $x = 10$

| $2(x + 3)$ | INPUT → [+ 3] → [x 2] → OUTPUT | = 26 |

Add 3 to the input then times by 2

If $x = 10$
$10 + 3 = 13..... 13 \times 2 = 26$
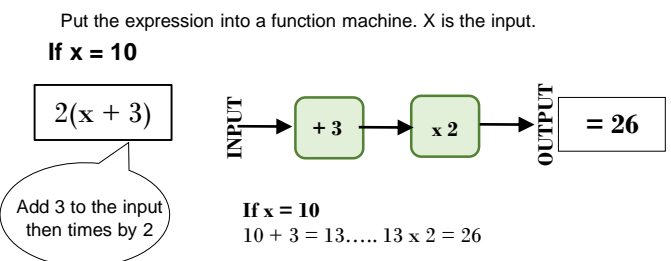
ALGEBRAIC THINKING...
Unit 5: Equality and Equivalence

What do I need to be able to do?

By the end of this unit you should be able to:
- Form and solve linear equations
- Understand like and unlike terms
- Simplify algebraic expressions by collecting like terms

Keywords

Equality: two expressions that have the same value
Equation: a mathematical statement that two things are equal
Equals: represented by '=' symbol. Means has the same value
Solution: the set or value that satisfies the equation
Solve: to find the solution.
Inverse: the operation that undoes what was done by the previous operation.
Term: a single number or variable
Like: variables that are the same are 'like'
Coefficient: the number in front of a variable e.g. 5x (5 is the coefficient, x is the variable)
Expression: a maths sentence with numbers or letters but no equals sign

Equality
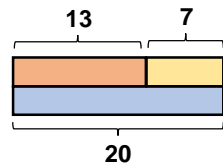
$$2 + 14 = 5 + 5 + 6$$

16   **16**

"Is equal to"

Saying it out loud sometimes helps you to understand equality

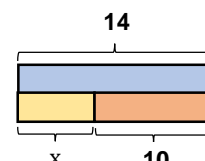The sum on the left has the same result as the sum on the right

Fact Families

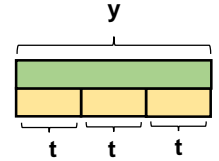Use a bar model to display the relationships between terms and numbers.

Model the information

13   **7**

20

Fact Family

$13 + 7 = 20 \quad 20 - 7 = 13$
$7 + 13 = 20 \quad 20 - 13 = 7$

14

x   **10**

$x + 10 = 14$
$10 + x = 14$

$14 - 10 = x$
$14 - x = 10$

y

t **t** **t**

$t + t + t = t$ $y - t - t = t$
y $y \div 3 = t$
$3 \times t = y$ $y \div t = 3$
$3t = y$

Solve one step equations (+/-)
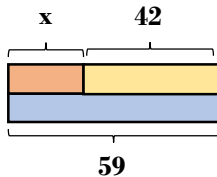
There is more to this than just spotting the answer
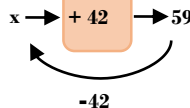
$$x + 42 = 59$$

$x + 42 = 59$
$42 + x = 59$

$59 - x = 42$
$59 - 42 = x$

x   **42**

59

Don't forget you know how to use function machines

$x \rightarrow +42 \rightarrow 59$
-42

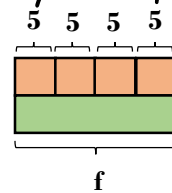### Solve one step equations (x/÷)

$$\frac{f}{4} = 5$$

$f \div 4 = 5$
$f \div 5 = 4$

$5 \times 4 = f$
$4 \times 5 = f$

5 5 5 5

f

Don't forget you know how to use function machines

$5 \rightarrow \times 4 \rightarrow f$
$\div 4$
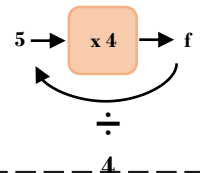
Like and unlike terms

Like terms are those whose variables are the same

$3 \heartsuit$ are **like terms**
the variable is the same

$3 \heartsuit$ are **unlike terms**
the variables are NOT the same

Examples and non-examples

Like terms	Un-like terms
y, 7y	y, 7x
$2x^2$, x^2	$2x^2$, $2c^2$
ab, 10ba	ab, 10a
5, -2	5, -2t

Note here ab and ba are commutative operations, so are still like terms

Equivalence

Check equivalence by substitution e.g. m=10

5m	**2 x 2m**	**7m – 3m**
5×10 $= 50$	$2 \times (2 \times 10)$ $= 2 \times 20$ $= 40$	$(7 \times 10) - (3 \times 10)$ $= 70 - 30$ $= 40$

Equivalent expressions

Repeat this with various values for m to check

5m

2 x 2m
4m

7m – 3m
4m
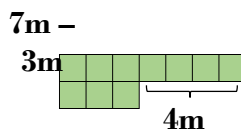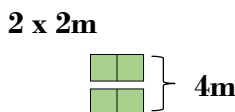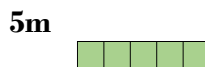
Collecting like terms

The $\equiv$ symbol means equivalent to.

It is used to identify equivalent expressions

Only **like terms** can be combined

$4x + 5b - 2x + 10b$

$4x \quad +5b \quad -2x \quad +10b$

$2x + 15b$

Common misconceptions

$2x + 3x^2 + 4x \equiv 6x + 3x^2$

Although they both have the x variable x^2 and x terms are un-like terms so can not be collected

ALGEBRAIC THINKING...
Unit 6: Sequences

What do I need to be able to do?

By the end of this unit you should be able to:
- Describe and continue both linear and non-linear sequences
- Explain term to term rules for linear sequence
- Find missing terms in a linear sequence
- Generate a sequence from term to term or position to term rules
- Recognise linear sequences and find the nth term

Keywords

Sequence: items or numbers put in a pre-decided order
Term: a single number or variable
Position: the place something is located
**Rule:** instructions that relate two variables
Linear: the difference between terms increases or decreases by the same value each time
Non-linear: the difference between terms increases or decreases in different amounts
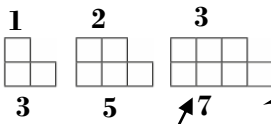**Difference:** the gap between two terms

Describe and continue a sequence diagrammatically

Count the number of circles or lines in each image

1   **3**   **5**   +2   +2

What will the next number be? Can you draw this?
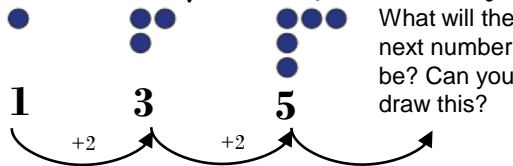
Sequence in a table and graphically

Position: the place in the sequence

1   **2**   **3**

3   **5**   **7**

"The **term** in **position** 3 has 7 squares"

Term: the number or variable
(the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2   +2

Graphically
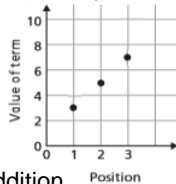
Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Sequences from algebraic rules

This is substitution!

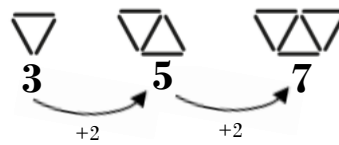This will be linear - note the single power of n. The values increase at a constant rate

This is not linear as there is a power for n

$$2n - 5$$ $$2n^2 - 5$$

Substitute the number of the term you are looking for in place of 'n'

e.g.
1st term = 2 x 1– 5 = -3
2nd term = 2 **x 2** – 5 = -1
100th term **=** 2 **x 100** – 5 = 195

Predict and check terms

3   **5**   **7**

+2   +2

CHECK – draw the next terms

9   **11**   **13**

Predictions:
Look at your pattern and consider how it will increase.

e.g. How many lines in pattern 6?
Prediction - 13
If it is increasing by 2 each time - in 3 more patterns there will be 6 more lines.

Linear and Non Linear Sequences

Linear Sequences – increase by adding or subtracting the same amount each time
Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci.
- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division.

Explain term-to-term rule

How you get from term to term

Try to explain this in full sentences not just with mathematical notation.
Use key maths language – doubles, halves, multiply by two, add four to the previous term etc.
To explain a whole sequence you need to include a term to begin at…

The next term is found by tripling the previous term. The sequence begins at 4.

4, 12, 36, 108…
x3   x3   x3

First term

Finding the algebraic rule

$$4n$$ **4, 8, 12, 16, 20…..**

This is the 4 times table

$$4n + 3$$ **7, 11, 15, 19, 22**

This has the same constant difference – but is 3 more than the original sequence

$$4n + 3$$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

OR it's the 0th term